

Math 223br: Rational Points on Varieties

Syllabus

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Logistics

We will have lectures each Tuesday and Thursday from 1:30-2:45PM in SC 110. Office hours will be after each lecture Thursday from 3-4PM in my office, SC 236. All course materials can be found on the course website: <https://ashvin-swaminathan.github.io/home/Math223br.html>.

Summary

In this course, we will give an introduction to the study of rational points on varieties. We will begin by discussing fields, culminating in the definition of the Brauer group. After introducing some of the algebraic geometry necessary to discuss rational points (e.g., varieties and morphisms), we will cover the theory of descent, as well as the theory of algebraic groups. We will conclude the course with a discussion of étale and fppf cohomology, as well as a treatment of cohomological obstructions to rational points.

Prerequisites

We will make use of concepts from the following subjects: abstract algebra, group theory, Galois theory, algebraic number theory, and algebraic geometry. It is highly recommended to have taken the equivalent of Math 123, 129, and 137; some familiarity with the concepts covered in Math 223a (particularly group cohomology) would not hurt. Undergraduates are welcome.

References

We will be following Bjorn Poonen's textbook *Rational Points on Varieties*, which you can find at the following link: <https://ashvin-swaminathan.github.io/home/Qpoints.pdf>. The hope is to cover Chapters 1-6 and 8.

Grading

Weekly homework assignments will make up 60% of the grade. Each week's homework will consist of three problems. Assignments will be posted to the course website on Thursday after lecture and will

be due on Tuesday at the start of lecture. If you have any questions, or if you require an extension or any other accommodation, please do not hesitate to email me, and I will be sure to respond promptly.

A final paper will count for the remaining 40% of the grade. This paper should be 7-10 pages in length and should discuss a topic related to the course material but not covered in lecture (this could be drawn from, e.g., Chapter 7 of the textbook, which I do not plan to cover in lecture). The final paper will be due on May 6 at 11:59pm; no extensions will be granted. I will be happy to answer any questions you have about the paper and to provide feedback on rough drafts.