

Math 295X Problem Set 3

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Problem 1

Prove that the Haar measure $dg = du(t^{-3}dt)d\theta(\lambda^{-1}d\lambda)$ on $\mathrm{GL}_2(\mathbb{R})$ is left-, right-, and inversion-invariant under the action of GL_2 ; i.e., prove that $d(\gamma g) = d(g\gamma) = d(g^{-1}) = dg$.

Problem 2

Let $v_i \in V(\mathbb{R})^{(i)}$ be a fixed element of discriminant $(-1)^i$. Compute the Jacobian determinant of the map $\mathrm{GL}_2(\mathbb{R}) \rightarrow V(\mathbb{R})^{(i)}$ defined in Iwasawa coordinates $(u, t, \theta, \lambda) \mapsto ut\theta\lambda \cdot v_i$. What is the Jacobian of the inverse map?¹

Problem 3

Prove that the number of divisors of a positive integer n is $\ll_\epsilon n^\epsilon$.

¹Recall that we showed that the action of $\mathrm{GL}_2(\mathbb{R})$ is transitive on $V(\mathbb{R})^{(i)}$.